B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMH5DSE21

(Probability & statistics)

Time: 3 Hours | Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: $6\times5 = 30$

(a) If two continuous random variables X and Y are connected by the relation $Y=g(X)$, then show that the probability density function of Y is given by $f_Y(y) = f_X(x)\left|\frac{dx}{dy}\right|$, where $y=g(x)$ is a continuously differentiable function of x which is strictly monotonic. [5]

(b) An experiment is to keep two balls into four boxes in such a way that each ball is equally likely to fall in a box. Let X denote the number of balls in the first box. Find the cumulative distribution function of X. Also find the mean and variance of X. [3+1+1]

(c) The lifetime (X) of electric bulbs in hours is supposed to be normally distributed as $\frac{1}{19\sqrt{2\pi}} e^{-\frac{(x-155)^2}{722}}$. [3+2]

(i) What is the probability that the lifetime of a bulb will be less than 117 hrs or more than 193 hrs.

(ii) What is the probability that the lifetime of a bulb will be between 117 and 193 hrs.

It is given that $\int_{-\infty}^{2} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 0.977.$

(d) A box contains 2^n tickets among which ${}^nc_i$ tickets bear the number i (i= 0,1,2,…,n). A group of m tickets are drawn. Find the expectation of the sum of their numbers. [5]

(e) The probability density function of a random variable X is given by [3+2]

$$f(x) = ae^{-ax},\ 0 < x < \infty, a > 0.$$

Obtain the moment generating function of X and hence find $E(X^n)$.

(f) The random variables X and Y are connected by the relation $3X + 5Y + 2 = 0$. Find the correlation coefficient between X and Y. [5]

(g) Estimate the parameter α of a continuous population having the density function $(1+\alpha)x^{\alpha}, 0 < x < 1$, by the method of maximum likelihood. Hence [5] find maximum likelihood estimate of α for the random sample of size 8 given by 0.2, 0.4, 0.8, 0.5, 0.7, 0.9, 0.8, 0.9.

(h) Let p denote the probability of getting a head when a coin is tossed once. Suppose that the hypothesis $H_0 : p=0.5$ is rejected in favour of $H_1 : p=0.6$. In 10 trials, if the outcome 'head' appears 7 or more times, calculate the probability of type-I and type-II errors. [3+2]

2. **Answer any three questions from the following:** 3×10 = 30

(a) (i) If the random variable X is distributed in the interval [- 1,1] with probability density function [5]

$$f(x) = \frac{3}{4}x^2, -1 \le x \le 0$$
$$= \frac{3}{4}, \quad 0 < x \le 1.$$

Find the distribution of $Y=|X|$.

(ii) A discrete random variable X takes the values 1,2,3.If $E(X)=1.7$ and $var(X)$ = 0.61, find the probability distribution of X. [5]

(b) (i) Prove that the joint distribution function $F(x,y)$ is monotonically non-decreasing function of both the variables x and y. [5]

(ii) A random point (X, Y) is uniformly distributed over a circular region $x^2 + y^2 < a^2$. Find the marginal distributions of X and Y, and the conditional distribution of Y given $X= x$,where $|x| < a$. [2+1+2]

(c) (i) Let X and Y be two independent normal random variables with means and standard deviations m_x, σ_x and m_y, σ_y respectively. Find the characteristic function of $Z= aX + bY$ (a, b are constants) and the distribution of Z. [3+2]

(ii) Let the random variables $X_1, X_2, .., X_n$ have the same distribution with finite mean m and finite standard deviation σ and if $X_1, X_2, .., X_n$ are mutually independent for all n, then prove that $\bar{X} \xrightarrow{in\,p} m$ as $n \to \infty$ where $\bar{X}$ is the mean of $X_1, X_2, .., X_n$. [5]

(d) (i) Obtain $100(1-\alpha)\%$ confidence interval for the parameter σ for the normal (μ, σ) population, when μ is known. [5]

(ii) In a ceramic industry the yield of first class material was known to have a mean 72.6. A new incentive bonus was declared and a subsequent sample of size 15 from the population gave a mean of 74.3 and standard deviation 1.9. Does this reasonably show that the bonus really helped in raising the average yield? Population is assumed to be normal or asymptotically normal and it is given that $P(t > 2.624) = 0.01$ for 14 degrees of freedom. [5]

B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMH5DSE22

(Portfolio Optimization)

Time: 3 Hours Full Marks: 60

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. Answer any six questions: 6×5 = 30

(a) Describe the three well known risk premiums. [5]

(b) Give the brief description about cash flow. [5]

(c) What are the risks of an *n*-security portfolio? [5]

(d) Write a short note about investment constraint. [5]

(e) Describe the procedure developed by Markowitz for choosing the optimal portfolio of risky assets. [5]

(f) Explain the nature of a risk-return indifference curve. [5]

(g) What is an efficient portfolio? Why is it called efficient? [4+1]

(h) What is the risk-free rate? How would you measure it? [3+2]

2. Answer any three questions: 10×3 = 30

(a) The stock of ABC Limited performs well relative to other stocks during recessionary periods. The stock of XYZ Limited, on the other hand, does well during growth periods. Both the stocks are currently selling for Rs 100 per share. You assess the rupee return (dividend plus price) of these stocks for the next year as follows: [3+3+4]

	Economic Condition		Stagnation	Recession
	High growth	Low growth		
Probability	0.3	0.4	0.2	0.1
Return on ABC's stock	100	110	120	140
Return on XYZ's stock	150	130	90	60

Calculate the expected return and standard deviation of investing:

(a) Rs 1,000 in the equity stock of ABC Limited

(b) Rs 1,000 in the equity stock of XYZ Limited

(c) Rs 500 each in the equity stock of ABC Limited and XYZ Limited.

(b) Mention the assumptions underlying the standard capital asset pricing model. Discuss the procedure commonly used in practice to test the CAPM. Despite its limitations, why is the CAPM widely used? [4+3+3]

(c) The returns of two assets under four possible states of nature are given below: [3+3+4]

State of nature	Probability	Return on asset 1	Return on asset 2
1	0.1	5%	0%
2	0.3	10%	8%
3	0.5	15%	18%
4	0.1	20%	26%

(a) What are the standard deviations of the return on asset 1 and asset 2?

(b) What is the covariance between the returns on assets 1 and 2?

(c) What is the coefficient of correlation between the returns on assets 1 and 2?

(d) The following table gives an analyst's expected return on two stocks for particular market returns: [2+2+3+3]

Market returns	Aggressive Stock	Defensive Stock
6%	2%	8%
20%	30%	16%

(a) What are the betas of the two stocks?

(b) What is the expected return on each stock if the market return is equally likely to be 6% or 20%?

(c) If the risk-free rate is 7% and the market return is equally likely to be 6% or 20%, what is the SML?

(d) What are the alphas of the two stocks?

(e) Define the return-generating process according to the APT. What is the equilibrium risk return relationship according to the APT? [5+5]

B.A/B.Sc 5th Semester (Honours) Examination, 2021 (CBCS)

Subject: Mathematics

Course: BMH5DSE23

(Boolean Algebra and Automata Theory)

Time -3 Hours **ull Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to write their answers in their own words as far as practicable.

[Notation and Symbols have their usual meaning]

1. **Answer any six questions:** $6 \times 5 = 30$

(a) (i) State duality principle. [2]

(ii) Every distributive lattice is modular. [3]

(b) (i) Define co-atom. [2]

(ii) In a finite Boolean algebra, prove that every non zero element is the sum of all the atoms dominated by it. [3]

(c) If f is a function of three Boolean variables x, y, z defined by $f(x,y,z) = xy + yz + zx$, express f in conjunctive normal form. [5]

(d) (i) Define modular lattice and give an example. [2]

(ii) A lattice is modular iff it satisfies $x \cup (y \cap (x \cup z)) = x \cup (z \cap (x \cup y))$. [3]

(e) A committee of three persons decides a proposal by a majority of votes. Each member can press a button to cast his vote. Design a simple circuit so that the light will glow when a majority of votes is cast in favour of the proposal. [5]

(f) Every regular language is recognized by a Turing machine. [5]

(g) (i) Give an example of regular language. [1]

(ii) If L and M are regular languages then prove that L ∩ M is also a regular language. [4]

(h) Show that, the balanced parenthesis language is not regular language. [5]

2. **Answer any three questions:** $10 \times 3 = 30$

(a) (i) In a Boolean Algebra B, prove that for a, b, c in B, [3]
$(a+b+c).(a+b+c').(a+b'+c).(a'+b+c) = a.b+b.c+c.a.$

(ii) Prove that (N, ≤) is a lattice where m ≤ n means m is a divisor of n. [4]

(iii) Prove that dual of a modular lattice is a modular lattice. [3]

(b) (i) Prove that the set of context-free languages is not closed under the operations of intersection and complement. [3]

(ii) Prove that the intersection of a regular language and a context-free language is context-free. [3]

(iii) Show that a context-free language is accepted by a nondeterministic turing machine. [4]

(c) (i) Given a context-free grammar G = (V, ∑, R, S), prove that there exists a pushdown automaton (PDA) M such that L(M) = L(G). [4]

(ii) Show that Post's Correspondence Problem is undecidable. [3]

(iii) Prove that every finite language is regular. [3]

(d) (i) Prove that the regular languages are a proper subset of the context-free languages. [3]

(ii) Prove that there are a countably infinite number of context-free languages. [3]

(iii) Using Pumping lemma, show that language $L = \{a^i b^j c^i d^j \ : \ i,j \geq 1\}$ is not a context-free language. [4]

(e) (i) State and prove De Morgan's laws in Boolean algebra. [3]

(ii) Prove that every Boolean algebra is atomic. Is the converse true? Justify your answer. [3+1]

(iii) Show that a lattice is distributive iff the following identity holds [3]
$(x \cap y) \cup (y \cap z) \cup (z \cap x) = (x \cup y) \cap (y \cup z) \cap (z \cup x)$.